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HEAT TRANSFER IN ALIGNED-FIELD MAGNETOHYDRODYNAMIC FLOW PAST A FLAT PLATE

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Abstract—Presented herein are studies of the steady-state heat-transfer phenomena in an aligned flow past a semi-infinite flat plate in which the flow velocity and magnetic field vectors far from the plate are parallel. A viscous, electrically conducting, incompressible fluid is used as the working medium. For simplicity, the physical properties are assumed constant and the electric field is taken to be zero. It is found that increasing the magnetic field increases the viscous, magnetic, and thermal boundary-layer thicknesses. The rate of heat transfer, however, is decreased with increasing magnetic field for Eckert number $Ek \leqslant 0$; while for $Ek > 0$, the opposite trend of heat-transfer rate is observed. Furthermore, for $Ek \leqslant 0$, the rate of heat transfer is higher at a larger Prandtl number; while for $Ek > 0$, lower values of Prandtl number render higher heat-transfer rates. The predicted heat-transfer results are outlined in this analysis.

NOMENCLATURE

$A(x, y)$,	magnetic stream function such that $\vec{H} = \nabla x(A\vec{i}_3)$;
b,	width of flat plate;
C,	specific heat;
Ek,	Eckert number $\left[\equiv \dfrac{V_\infty^2}{2C(t_w - t_\infty)}\right]$;
$f(\eta)$,	defined as $\Psi(x, y)/\sqrt{(\nu V_\infty x)}$;
$g(\eta)$,	defined as $A(x, y)/H_0\sqrt{(\nu x/V_\infty)}$;
H,	magnetic field;
$\vec{i}_3$,	unit vector in z-direction;
l,	length of flat plate;
Pm,	magnetic Prandtl number $(\equiv \bar{\mu}\sigma\nu)$;
Pr,	Prandtl number $(\equiv \mu C/\kappa)$;
q,	heat flux density at fluid–solid boundary;
Re,	Reynolds number $(\equiv V_\infty l/\nu)$;
Re_x,	local Reynolds number $(\equiv V_\infty x/\nu)$;
S,	magnetic force number $(\equiv \bar{\mu}H_0^2/\rho V_\infty^2)$;
$T(x, y)$, $t(\eta)$,	temperature;
v,	fluid velocity;
x, y,	coordinate axes along and normal to flat plate.

Greek symbols

α,	thermal diffusivity $(\equiv \kappa/\rho C)$;
δ_H,	magnetic boundary-layer thickness;
δ_t,	thermal boundary-layer thickness;
δ_v,	viscous or velocity boundary-layer thickness;
η,	independent variable $[= y/2\sqrt{(V_\infty/\nu x)}]$;
$\theta(\eta)$,	solution to homogeneous equation (20) $\left(= \dfrac{t(\eta) - t_w}{t_\infty - t_w}\right)$;
$\theta_r(\eta)$,	temperature parameter $\left(= \dfrac{t(\eta) - t_\infty}{V_\infty^2/2C}\right)$;

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κ, thermal conductivity;
μ, dynamic viscosity;
$\bar{\mu}$, magnetic permeability;
ν, kinematic viscosity ($\equiv \mu/\rho$);
ρ, fluid density;
σ, electrical conductivity;
Φ, function given by equation (14);
$\Psi(x, y)$, fluid stream function such that $\vec{v} = \nabla x(\Psi \vec{i}_3)$;
$\psi(\eta)$, temperature profile $\left(= \dfrac{t(\eta) - t_\infty}{t_w - t_\infty}\right)$.

Subscripts
0, applied condition;
∞, free stream condition;
w, condition at fluid–solid boundary;
x, y, components in x- and y-co-ordinate axes.

Superscripts
$', '', '''$, first, second and third derivatives with respect to η.

INTRODUCTION

THE PROBLEM investigated is the aligned flow past a semi-infinite flat plate in which the flow velocity and magnetic field vectors far from the plate are parallel. The momentum transfer phenomena of the aforementioned flows have been studied by Greenspan and Carrier [1]. An initial attempt at solution of the heat-transfer phenomena connected with such a problem has been made [2], but unfortunately this earlier treatment is inadequate due to several errors. A more thorough study of the aforementioned problem, which takes into consideration both viscous and ohmic heating, is the attempt of this analysis.

The solutions to the governing equations of the velocity and magnetic fields are utilized to render solutions to the energy equation by direct numerical integration. At the fluid–solid boundary, either a uniform wall temperature or a constant specific heat flux is prescribed. Of particular interest is the case of a thermally insulated (zero heat flux) plate. This leads to the determination of the recovery factor and the recovery temperature and is used in connection with the uniform wall temperature case to determine the direction of heat transfer. In this study, temperature distribution, development of thermal boundary-layer thickness, heat flux across the fluid–solid boundary and the Nusselt number are theoretically investigated.

GOVERNING EQUATIONS AND SIMILARITY TRANSFORMATIONS

For the MHD flow past a flat plate in the presence of a parallel magnetic field, by assuming a viscous, electrically conducting, incompressible fluid with constant physical properties, the leading order boundary-layer equations for the velocity, magnetic and temperature fields, in the absence of the electric field, are [1, 2]:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \tag{1}$$

$$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} = 0 \tag{2}$$

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \nu \frac{\partial^2 v_x}{\partial y^2} + \frac{\bar{\mu}}{\rho}\left(H_x \frac{\partial H_x}{\partial x} + H_y \frac{\partial H_x}{\partial y}\right) \tag{3}$$

$$-\frac{\partial H_x}{\partial y} = \sigma\bar{\mu}(v_x H_y - v_y H_x) \tag{4}$$

$$v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C}\left(\frac{\partial v_x}{\partial y}\right)^2 + \frac{1}{\rho C \sigma}\left(\frac{\partial H_x}{\partial y}\right)^2, \tag{5}$$

with the following boundary conditions:

$$y = 0: \quad v_x = v_y = 0, \quad H_y = 0, \quad T = T_w \text{ or } \frac{\partial T}{\partial y} = -\frac{q_0}{\kappa} \text{ or } \frac{\partial T}{\partial y} = 0 \tag{6}$$

$$y \to \infty: \quad v_x = V_\infty, \quad H_x = H_0, \quad T = T_\infty.$$

In the energy equation, both the viscous dissipation $\mu(\partial v_x/\partial y)^2$ and the ohmic heating $1/\sigma\,(\partial H_x/\partial y)^2$ terms are included. It is thus noted that equations (3) and (4) have first to be simultaneously solved prior to the solution of the heat-transfer problem.

The nonlinearity of the momentum equation makes it difficult to obtain a closed mathematical solution to the problem. However, by invoking the following transformations:

$$\Psi(x, y) = (\sqrt{\nu V_\infty x})\, f(\eta),$$

$$A(x, y) = \sqrt{\left(\frac{\nu x}{V_\infty}\right)}\, H_0 g(\eta),$$

$$T(x, y) = t(\eta),$$

and

$$\eta = \frac{y}{2}\sqrt{\frac{V_\infty}{\nu x}},$$

equations (3), (4), and (5) become

$$f''' + ff'' - Sgg'' = 0 \tag{7}$$

$$g'' + Pm(fg' - gf') = 0 \tag{8}$$

$$t'' + Pr\, ft' + \frac{V_\infty^2}{4C} Pr \left[(f'')^2 + \frac{S}{Pm}(g'')^2\right] = 0, \tag{9}$$

where $S \equiv \bar{\mu}H_0^2/\rho V_\infty^2$ is the ratio of the electromagnetic to the inertia forces, sometimes referred to as the magnetic force number; $Pm \equiv \bar{\mu}\sigma\nu$ is the magnetic Prandtl number; and $Pr \equiv \mu C/\kappa$ is the ordinary Prandtl number. The corresponding transformed boundary conditions are:

$$f(0) = f'(0) = 0, \qquad f'(\infty) = 2$$

$$g(0) = 0, \qquad g'(\infty) = 2$$

$$t'(0) = -\frac{2q_0'}{\kappa}\sqrt{\left(\frac{\nu x}{V_\infty}\right)} \quad \text{or} \quad t'(0) = 0 \quad \text{or} \quad t(0) = t_w, \quad t(\infty) = t_\infty. \tag{10}$$

When $S = 0$, which is the nonmagnetic case, equation (7) reverts to the classical Blasius equation for a hydrodynamic flow. Under this situation, equation (13) of the magnetic field will no longer be needed and can thus be dropped. It is further noted that when $Pm = 0$, such as is the case of zero electrical conductivity, equation (8) reduces to $g'' = 0$ which, in turn, renders equation (7) of the velocity field independent of the magnetic field. This is physically the case since, for $\sigma = 0$, no interaction between the two fields can occur.

NUMERICAL SOLUTION TO EQUATIONS OF MOTION AND MAGNETIC FIELD

In determining the functions $f(\eta)$ and $g(\eta)$, which satisfy the two coupled differential equations and the boundary conditions, equations (7), (8), and (10), respectively, a numerical method was used. The IBM 7094 computer was used for this computation. The values of the parameter S used in the calculation are consistent with the existence of the steady-state solution of "super-Alfvén" flow; by this we mean a flow in which the free stream velocity V_∞ is larger than the Alfvén wave speed $(H_0\sqrt{\bar{\mu}/\rho})$, i.e. $S < 1$. For "sub-Alfvén" flow, $S > 1$, any disturbance within the boundary layer can propagate upstream by means of the Alfvén waves, thereby making the flow phenomena unstable. It has been proven [3] that for $0 < S < 1$ and $Pm < 1$, numerical procedures yield two different solutions. Further evidence of this nonuniqueness is given by Stewartson and Wilson [4]. Our results correspond to the upper branch of the dual solutions. The lower branch is ignored in this analysis as no physical significance has been attached to it.

Figures 1 and 2 depict some typical variations of f, g, and their derivatives with η for $S = 0$ and 0·5, and $Pm = 0$, 0·1, and 1·0. It is observed that, as the value of S increases, both f' and g'

decrease everywhere within the boundary layer. Under these circumstances, the normal component of the velocity as well as the magnetic field distributions assume greater asymptotic values than the nonmagnetic case. The viscous as well as the magnetic boundary-layer thickness, however, increase with increasing S and with decreasing Pm as shown in Fig. 3. For values of $Pm \simeq 1{\cdot}0$, the two boundary-layer thicknesses almost coincide; but for values of $Pm < 1{\cdot}0$, the magnetic boundary-layer thickness is usually larger than the viscous boundary-layer thickness at small values of S, and merges with the latter at large values of S.

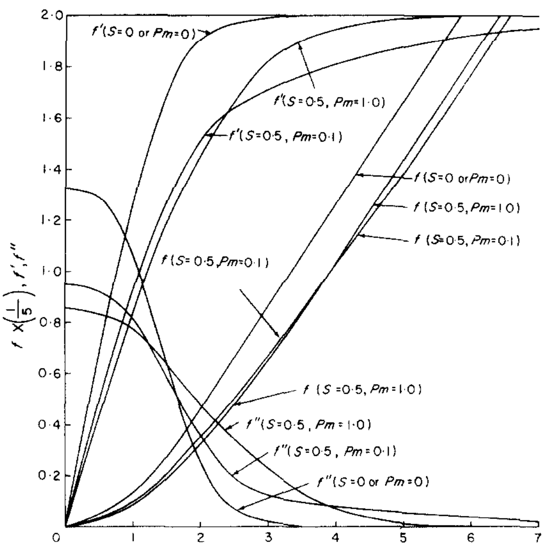

FIG. 1. Variations of f, f', and f'' with η for $S = 0$, $0{\cdot}5$; $Pm = 0$, $0{\cdot}1$, $1{\cdot}0$.

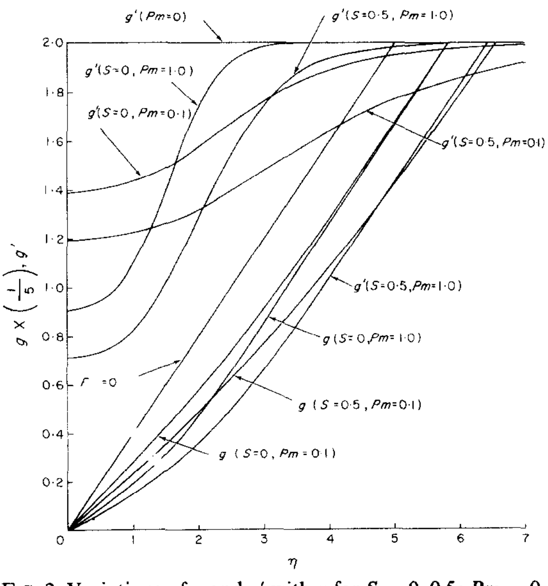

FIG. 2. Variations of g and g' with η for $S = 0$, $0{\cdot}5$; $Pm = 0$, $0{\cdot}1$, $1{\cdot}0$.

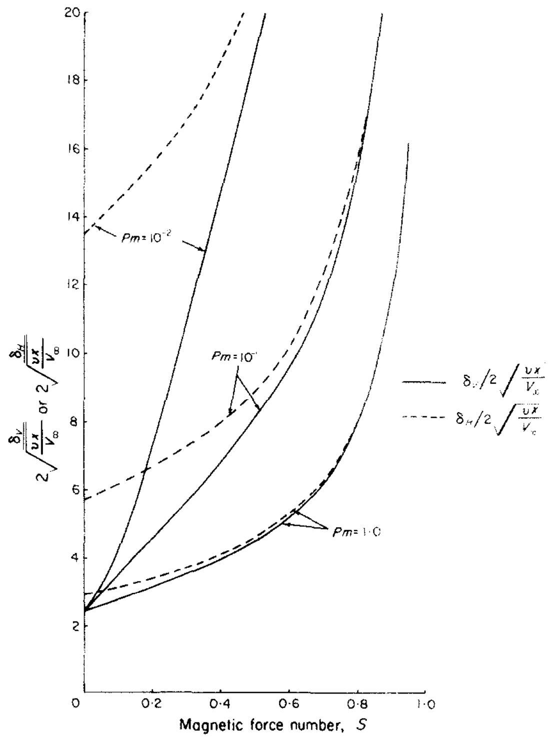

FIG. 3. Development of velocity and magnetic boundary-layer thicknesses.

SOLUTIONS TO THE ENERGY EQUATION

The energy equation which describes heat transfer in a laminar, steady boundary layer, including the effects of viscous and ohmic heating, is represented by equation (9). In this analysis two situations are considered, in which the fluid–solid boundary is either maintained with prescribed heat flux or at uniform wall temperature.

Prescribed heat flux at fluid–solid boundary

In terms of a dimensionless temperature parameter

$$\theta_r(\eta) \equiv \frac{t(\eta) - t_\infty}{(V_\infty^2/2C)}$$

which expresses the temperature field within the boundary layer, the energy equation (9) becomes

$$\theta_r'' + Pr\, f\theta_r' + \tfrac{1}{2}Pr\left[(f'')^2 + \frac{S}{Pm}(g'')^2\right] = 0. \quad (11)$$

The boundary conditions for this case are

$$\theta_r'(0) = \frac{4C}{V_\infty^2}\left(\frac{q_0}{\kappa}\right)\Big/\sqrt{\left(\frac{\nu x}{V_\infty}\right)} \quad \text{and} \quad \theta_r(\infty) = 0.$$

This second-order, inhomogeneous, linear differential equation for the temperature parameter $\theta_r(\eta)$ can be solved numerically once the functional values of $f(\eta)$ and $g(\eta)$ are known. A solution of equation (11) can be obtained, for instance, by the method of variation of coefficients and results in

$$\theta_r(\eta) = \frac{Pr}{2}\int_0^\infty \Phi\, d\eta - \frac{Pr}{2}\int_0^\eta \Phi\, d\eta + \int_\eta^\infty \frac{4Cq_0}{\kappa V_\infty^2} \times \sqrt{\left(\frac{\nu x}{V_\infty}\right)} \exp\left(-\int_0^\eta Pr\, f\, d\eta\right) d\eta, \quad (12)$$

with

$$\Phi = \exp\left(-\int_0^\eta Pr\, f\, d\eta\right)\left\{\int_0^\eta \left[(f'')^2 + \frac{S}{Pm}(g'')^2\right] \exp\left(\int_0^\eta Pr\, f\, d\eta\right) d\eta\right\}. \quad (13)$$

The insulated boundary is a special case of the prescribed heat flux at the fluid–solid boundary discussed above, in that the boundary conditions $\theta_r'(0) = 0$ and $\theta_r(\infty) = 0$. The solution to equation (11), subject to these conditions, can be obtained from equation (12) by setting $q_0 = 0$, i.e.

$$\theta_r(\eta) = \frac{Pr}{2}\int_\eta^\infty \Phi\, d\eta, \quad (14)$$

in which the function Φ is given by equation (13). The temperature that the fluid–solid boundary assumes is its recovery temperature, which is given by

$$t_r = t_\infty + \theta_r(0)\left(\frac{V_\infty^2}{2C}\right), \quad (15)$$

and the value of the parameter $\theta_r(0)$ is the recovery factor given by

$$\theta_r(0) = \frac{Pr}{2}\int_0^\infty \Phi\, d\eta. \quad (16)$$

The distribution of the dimensionless temperature parameter $\theta_r(\eta)$ is shown in Fig. 4. The effect of increasing S is a decrease in temperature in the region near the fluid–solid boundary, but an increase in temperature in the regions far from the boundary. The thermal boundary-layer thickness is thus increased with increasing S. Values for the recovery factor $\theta_r(0)$, as shown in Fig. 5, decrease with increasing S and increasing Pm. The recovery temperature t_r thus decreases with increasing S and Pm since an increase in either S or Pm reduces the wall friction, thereby reducing the viscous heating. When $S = 0$, the recovery factor varies with the square root of the Prandtl number, which is consistent with that obtained in classical hydrodynamic flow over a flat plate [5].

Uniform temperature at fluid–solid boundary

We next consider heat transfer at the fluid–solid boundary when its temperature is kept constant by cooling or heating at a value t_w, which is different from the recovery temperature t_r given by equation (15). The general solution of equation (11) can be composed of a particular solution of the inhomogeneous equation plus a complimentary solution of the

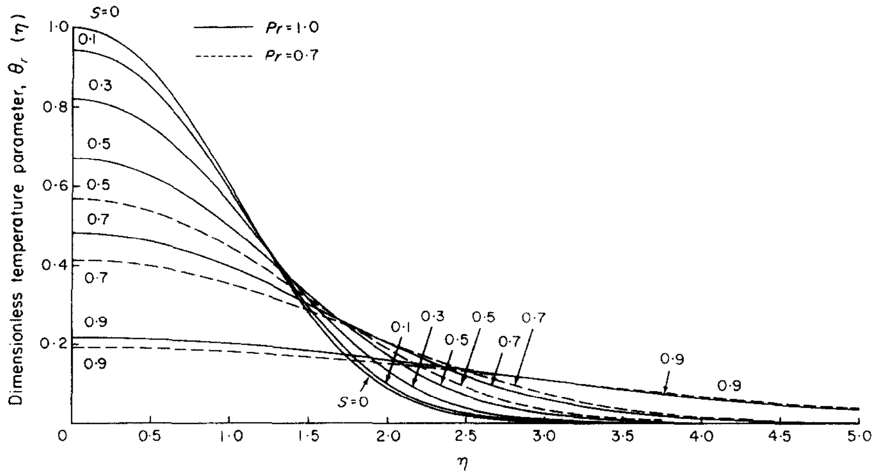

FIG. 4. Distribution of dimensionless temperature parameter $\theta_r(\eta)$ for flow over an adiabatic flat plate, $Pm = 1{\cdot}0$ and $Pr = 0{\cdot}7$, $1{\cdot}0$.

corresponding homogeneous equation. A particular solution of the inhomogeneous equation has been found in equation (14). The corresponding homogeneous equation is

$$\theta'' + Pr\, f\theta' = 0, \tag{17}$$

where

$$\theta(\eta) \equiv \frac{t - t_w}{t_\infty - t_w},$$

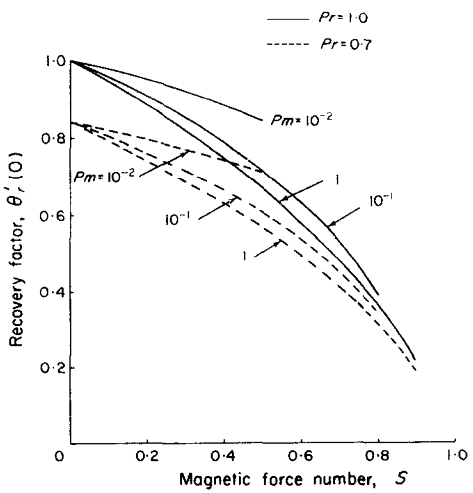

FIG. 5. Variation of the recovery factor with the magnetic force number, $Pm = 10^{-2}$, 10^{-1}, 1 and $Pr = 0{\cdot}7$, $1{\cdot}0$.

with the following boundary conditions:

$$\theta(0) = 0 \quad \text{and} \quad \theta(\infty) = 1. \tag{18}$$

The general solution of equation (17), subject to equation (18), is

$$\theta(\eta) = \frac{\int_0^\eta \exp\left(-\int_0^\eta Pr\, f\, d\eta\right) d\eta}{\int_0^\infty \exp\left(-\int_0^\eta Pr\, f\, d\eta\right) d\eta}. \tag{19}$$

The temperature field for the high velocity boundary-layer flow past a flat plate can be written as [5]

$$t(\eta) = \theta_r(\eta)\frac{V_\infty^2}{2C} + (t_w - t_r)[1 - \theta(\eta)] + t_\infty, \tag{20a}$$

or

$$\psi(\eta) \equiv \frac{t(\eta) - t_\infty}{t_w - t_\infty} = Ek\,\theta_r(\eta) + [1 - Ek\,\theta_r(0)][1 - \theta(\eta)], \tag{20b}$$

in which $\theta_r(\eta)$ is given by equation (14), $\theta_r(0)$ by equation (16), $\theta(\eta)$ by equation (19), and $Ek \equiv V_\infty^2/2C(t_w - t_\infty)$ is the Eckert number.

Temperature profiles as described by equation (20b) are shown in Figs. 6 and 7. Three observations are made here. First, when $Ek > 0$, $\psi(\eta)$ and, hence, the temperature field decrease with increasing S in the region close

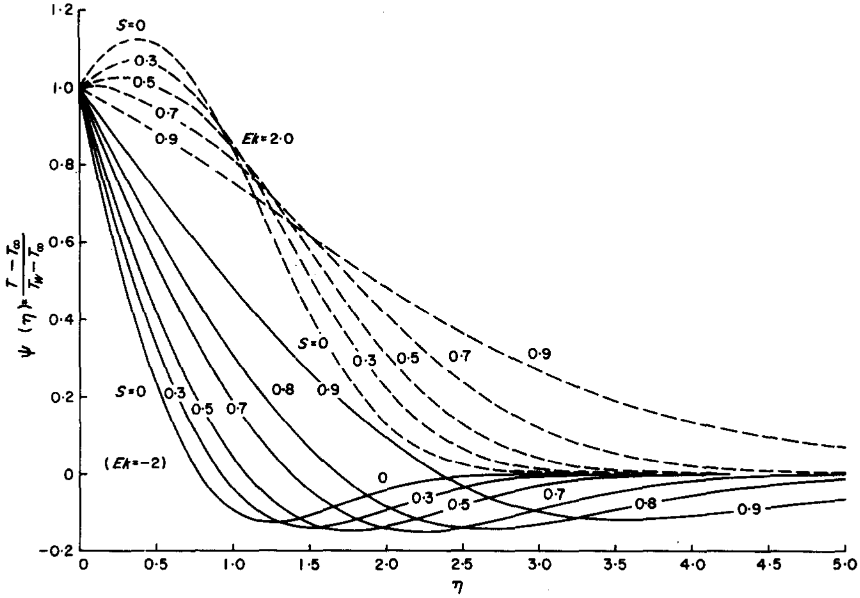

FIG. 6. Temperature profile for flow past an isothermal plate, $Pm = 1{\cdot}0$, $Pr = 1{\cdot}0$.

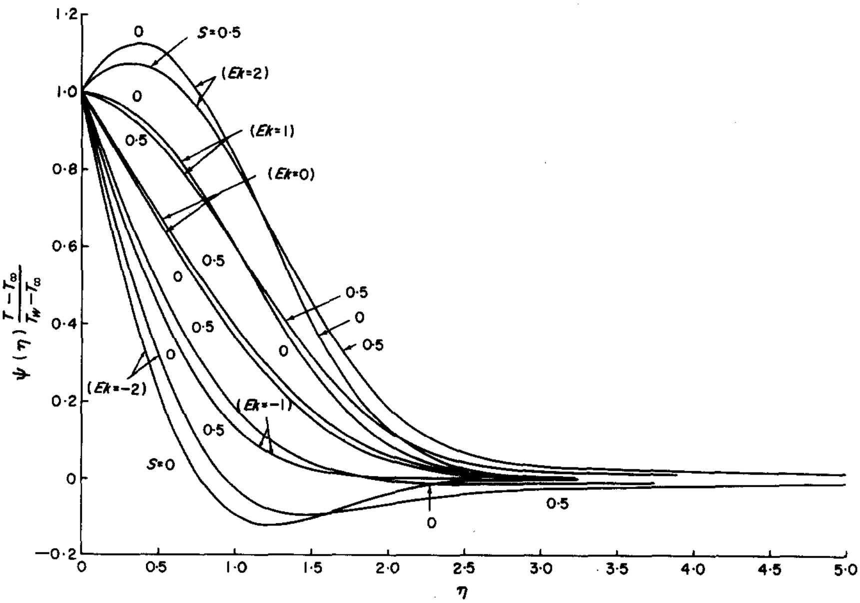

FIG. 7. Temperature profile for flow past an isothermal plate, $Pm = 10^{-2}$, $Pr = 1$.

to the plate, whereas ψ increases with increasing S in regions far from the plate. The reverse trend is seen when $Ek < 0$. Secondly, the effect of increasing Pm, when $Ek > 0$, is the same as that of S, in that ψ decreases with increasing Pm near the plate, but increases in regions far from the plate as Pm increases. When $Ek < 0$, the temperature function ψ is seen to increase with increasing Pm. And thirdly, the temperature function ψ may be greater than unity or less than zero in some region within the boundary layer depending, respectively, on whether $Ek > 1$ or $Ek < (-1)$.

The development of the thermal boundary-layer thickness with magnetic force number is shown in Fig. 8 for $Pm = 10^{-2}$, 10^{-1}, 1; $Ek = -2, -1, 0, 1, 2$; and $Pr = 1$.

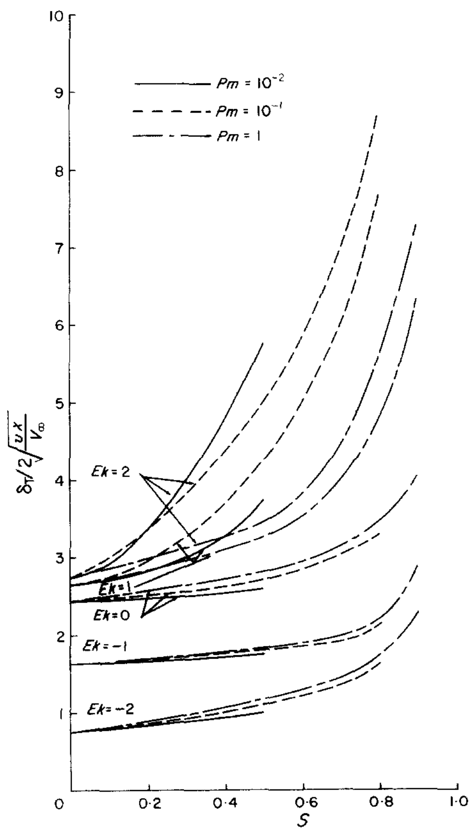

FIG. 8. Development of thermal boundary-layer thickness for $Pr = 1$.

Of special importance to engineering applications is the rate of heat transfer per unit area for the high-speed flow problem at the fluid–solid boundary. This is given by

$$q = -\kappa \frac{\partial T}{\partial y}\bigg|_{y=0} = -\kappa \frac{\mathrm{d}t}{\mathrm{d}\eta}\bigg|_{\eta=0} \frac{\partial \eta}{\partial y}\bigg|_{y=0}$$
$$= \frac{\kappa}{2}\sqrt{\left(\frac{V_\infty}{\nu x}\right)}\,\theta'(0)\,(t_w - t_r), \qquad (21a)$$

since $\theta_r'(0) = 0$ for an adiabatic plate. Here $\theta'(0) > 0$, and the direction of heat flow is governed primarily by the sign of $(t_w - t_r)$, the latter quantity being the driving force in this problem. When $(t_w - t_r) < 0$, $q < 0$, heat energy is being transferred from the fluid to the plate. When $(t_w - t_r) > 0$, $q > 0$, heat flow takes place in the reverse direction. Applying equation (15) and the definition of the Eckert number to the above heat flux expression results in

$$q = \frac{\kappa}{2}\sqrt{\left(\frac{V_\infty}{\nu x}\right)}\,\theta'(0)\,(t_w - t_\infty)\,[1 - Ek\,\theta_r(0)]. \qquad (21b)$$

The following remarks can, therefore, be made:

(a) If $Ek < 0$, i.e. $(t_w - t_\infty) < 0$, then $q > 0$, the direction of heat flow is from the fluid to the plate.
(b) If $Ek = 0$, the recovery temperature is equal to the free stream temperature, and the direction of heat flow is mainly determined by the sign of $(t_w - t_\infty)$. When $(t_w - t_\infty) > 0$, heat energy is being transferred from the plate surface to the fluid, and in the reverse direction when $(t_w - t_\infty) < 0$. When $(t_w - t_\infty) = 0$, no net heat transfer takes place.
(c) If $Ek > 0$, i.e. $(t_w - t_\infty) > 0$, the following situations could arise: (i) when $[1 - Ek\,\theta_r(0)] > 0$, heat transfer takes place from the plate surface to the fluid, (ii) when $[1 - Ek\,\theta_r(0)] = 0$, no net heat transfer takes place, and (iii) when $[1 - Ek\,\theta_r(0)] < 0$, heat energy is being transferred from the fluid to the plate.

The total heat flux across one surface of the semi-infinite flat plate of length l and width b can be found from

$$Q = \int_0^l q(x) b \,\mathrm{d}x$$

to be

$$Q^* \equiv \frac{Q}{\kappa b (t_w - t_\infty)\sqrt{(Re)}} = \theta'(0)\,[1 - Ek\,\theta_r(0)]. \qquad (22)$$

Variations of Q^* with the magnetic force number

S for various values of Ek, Pm and Pr are shown in Figs. 9(a, b). Generally Q^* decreases with increasing S for $Ek \leqslant 0$, and increases with increasing S for $Ek > 0$. The rate of decrease of the increase of Q^* with S, however, is less prominent at lower values of magnetic Prandtl number than at higher values. Also, for specific values of S and Pm, Q^* is seen to increase with decreasing Prandtl number when $Ek > 0$, but it decreases with decreasing Prandtl number when $Ek < 0$.

If one introduces Newton's law of cooling as $q = h(t_w - t_r)$, where h is the heat-transfer coefficient, one can then equate this to equation (21a), from which a dimensionless local Nusselt number is formed

$$Nu_x \equiv \frac{hx}{\kappa} = \tfrac{1}{2}\theta'(0)\sqrt{(Re_x)}. \qquad (23)$$

Figure 10 shows the variations of $Nu_x/\sqrt{(Re_x)}$ with S for various values of Pr and Pm. This ratio decreases with increasing S or Pm, akin to the results for the coefficient of friction [1].

DISCUSSION

While it is established that both the velocity and magnetic boundary-layer thicknesses increase with increasing magnetic force number S and decreasing magnetic Prandtl number Pm (Fig. 3), the same remark can also be applied to the thermal boundary-layer thickness at large Eckert number ($Ek > 1$) (Fig. 8). For $Ek \leqslant 1$, though the thermal boundary-layer thickness still increases with increasing S, its dependence on Pm depends partly on the Eckert number. In all cases, however, the thermal boundary-layer thickness increases with increasing Eckert number, and for Ek as large as 2 and at any specific value of magnetic Prandtl number less than unity, it is consistently less than the velocity, and hence, the magnetic boundary-layer thicknesses.

As it was previously observed, [2] the rate of heat transfer decreases with increasing magnetic field for $Ek \leqslant 0$, while for $Ek > 0$, the opposite trend is observed [Figs. 9(a, b)]. It is also interesting to note that, for $Ek \leqslant 0$, the rate of

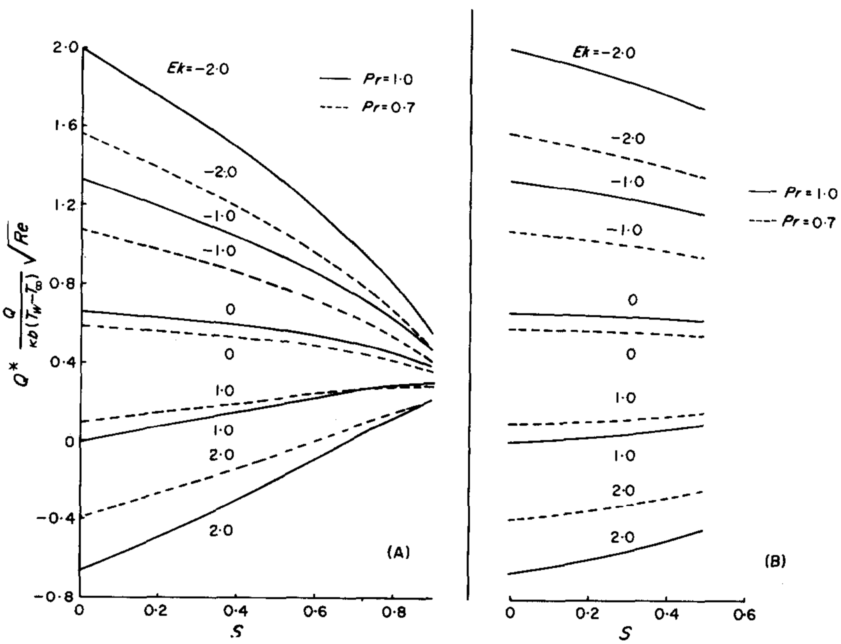

FIG. 9(a). Variation of heat-transfer rates with S, Pm = 1·0 and Pr = 0·7, 1·0.

FIG. 9(b). Variation of heat-transfer rates with S, $Pm = 10^{-2}$, Pr = 0·7, 1·0.

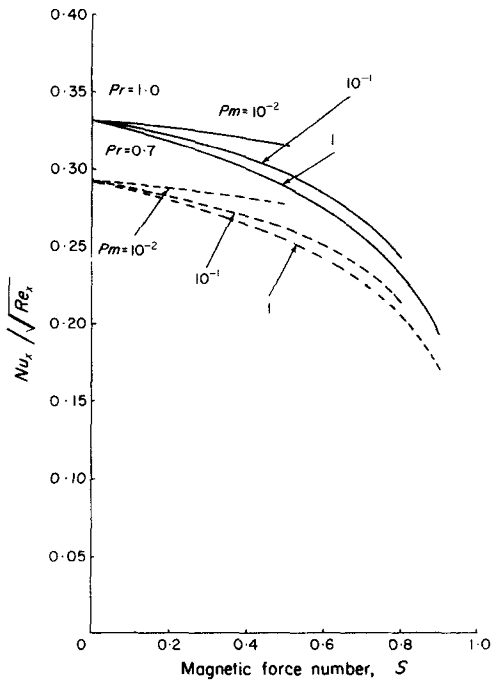

FIG. 10. Variations of $Nu_x/\sqrt{(Re_x)}$ with S, $Pm = 10^{-2}, 10^{-1}$, 1; $Pr = 0{\cdot}7$, 1·0.

heat transfer is higher at larger Prandtl number, while for $Ek > 0$, lower values of Prandtl number render higher heat-transfer rates.

For high speed flow, the direction of heat transfer for flow past a uniform temperature flat plate is determined by the difference between the wall temperature and its recovery temperature. It is thus felt that the analysis regarding this aspect of heat transfer, taking into consideration both the viscous and ohmic heating, is quite adequate.

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Résumé—On présente ici des études des phénomènes de transport de chaleur en régime permanent dans un écoulement le long d'une plaque plane semi-infinie dans lequel les vecteurs vitesse d'écoulement et champ magnétique loin de la plaque sont parallèles. Le milieu de travail employé est un fluide incompressible, conducteur de l'électricité et visqueux. Pour simplifier, les propriétés physiques sont supposées constantes et le champ électrique est pris égal à zéro. On trouve que l'augmentation du champ magnétique fait croître les épaisseurs des couches limites visqueuse, magnétique et thermique. Cependant, le taux de transport de chaleur décroît lorsque le champ magnétique croît pour un nombre d'Eckert $Ek \leqslant 0$; tandis que pour $Ek > 0$, on observe une tendance contraire pour le taux de transport de chaleur.

De plus, lorsque $Ek \leqslant 0$, le taux de transport de chaleur est plus élevé pour des nombres de Prandtl plus grands; tandis que pour $Ek > 0$, des valeurs plus faibles du nombre de Prandtl rendent plus élevés les taux de transport de chaleur. Dans cette analyse, on donne un aperçu des résultats de transport de chaleur prévus.

Zusammenfassung—Es wird von Untersuchungen berichtet über den stationären Wärmeübergang in einer geradlinigen Strömung entlang einer halbunendlichen ebenen Platte wobei die Vektoren von Strömungsgeschwindigkeit und Magnetfeld in grosser Entfernung von der Platte parallel sind. Als Arbeitsmedium dient eine zähe, elektrisch leitende inkompressible Flüssigkeit. Vereinfachend sind die physikalischen Stoffwerte als konstant und das elektrische Feld zu Null angenommen. Es zeigt sich, dass eine Vergrösserung des Magnetfeldes zu einer Verdickung der zähen, magnetischen und thermischen Grenzschicht führt. Der Wärmeübergang wird andererseits durch Vergrösserung des Magnetfeldes für Eckert–Zahlen $Ek \leqslant 0$ herabgesetzt, während für $Ek > 0$ eine Vergrösserung beobachtet wird. Weiterhin ist für $Ek \leqslant 0$ bei grösseren Prandtl-Zahlen der Wärmeübergang grösser, während er bei $Ek > 0$ für kleinere Werte der Prandtl-Zahl die grösseren Werte für den Wärmeübergang liefert. Die berechneten Wärmeübergangsergebnisse sind in dieser Analyse dargestellt.

Аннотация—Изложены результаты исследования явлений стационарного теплообмена полубесконечной плоской пластины, обтекаемой продольным потоком, в котором векторы скорости и магнитного поля параллельны на большом расстоянии от пластины.

В качестве рабочей среды используется вязкая электропроводная несжимаемая жидкость. Для упрощения физические свойства считаются постоянными, а электрическое поле принимается равным нулю. Установлено, что при увеличении магнитного поля увеличивается толщина вязкого магнитного и теплового пограничного слоев. Однако, при увеличении магнитного поля скорость теплообмена уменьшается для числа Эккерта $Ek < 0$; тогда как для $Ek > 0$ наблюдается обратное соотношение. Кроме того, для $Ek < 0$ скорость теплообмена выше при больших числах Прандтля, тогда как для $Ek > 0$, более низкие числа Прандтля соответствуют более высоким скоростям переноса. Приведены некоторые расчетные данные по теплообмену.